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Analytical modelling of Plane Wave Obliquely Incident on Two Dielectric media

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Abstract
In this paper an accurate analysis of the behavior of an arbitrary polarized plane wave on a dielectric-dielectric interface is examined. The reflection and refraction spectra for planar interface separating two dialectic media have been investigated by imposing suitable boundary condition. The reflection and refraction coefficients, reflectance and the variation of refracted angle with angle of incidence are investigated. The variation of the structure parameters such as constitutive parameters and incident angle leads to dramatic changes of the behavior of plane wave. This study presents detailed results for dielectric-dielectric interface, and discus the key design parameters and compression between dielectric materials.

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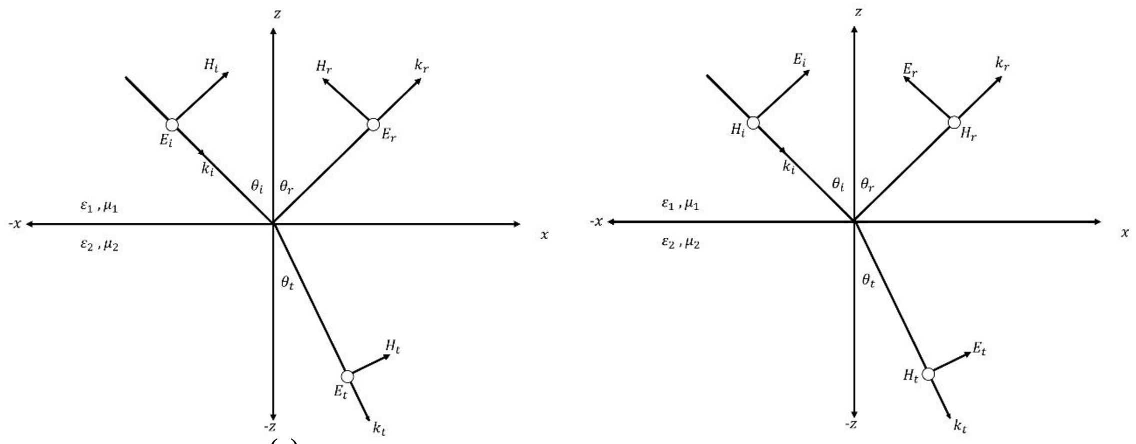
I. Introduction

When a plane wave is obliquely incident on a plane interface between two media, the formulation of boundary conditions becomes more complex than when incidence isnormal. Certainly, again a part of the incident energy is reflected back into first medium and a part is transmitted into second medium[1]. With respect to the plane on incidence, plane wave impinges upon an interface can be decomposed to a S-polarized andP-polarized wave [2]. In S-polarized case, the electrical fields are normal to the plane of incidence, hence we refer to this polarization as transverse electric or TE-polarization. On the other hand, when the magnetic fields are transvers to the plane of incidence we refer to this polarization as transvers magnetic or TM-polarization [3].

In this paper, MATLAB code is developed in order to clarify the refraction and reflection of a vector field at a planar interfaces separating two dielectric media.The analysis is mainly focused on the behavior of plane wave where different investigationshave been carried out to explore the effect of different parameters of the structure on transmitted and reflected power. Parameter such as the polarization type, angle of incidence and refractive indices have been varied and sets of different results have been presented. The well-known Snell's low is given by 'phase shifting' the field at the interface. Another interesting limiting incident angle is the polarization or Brewster's angle at which the incident power is totally transmitted which occurs only for p-polarization. Therefore, the interface may be regarded as polarizer and hence the name attributed to this angle is the polarization angle. This paper is organized as following. Following this introduction, a brief mathematical analysis is given in section 2. The simulation results are presented in section 3. Finally, conclusions are drawn.

II. Analysis

2.1 Maxwell's equation and wave equation
Starting from Maxwell's equation and for 2D structures in x-z plane and for oblique incidence, the electric field may lie on this plane (TE-mode) or perpendicular to it (TM-mode)



(a)

(b)

Fig.1: planar interface between two media with polarization (a) Perpendicular Polarization (TE – mode), (b) Parallel Polarization (TM – mode).

the Maxwell's equations in their differential form for electromagnetic propagation are written as [4]:

$$\nabla D = \rho \tag{1a}$$

$$\nabla B = 0 \tag{1b}$$

$$\nabla \times E = M - \partial B/\partial t \tag{1c}$$

$$\nabla \times H = J - \partial D/\partial t \tag{1d}$$

Where E is the vectorial electric field in V/m, H is the vectorial magnetic field in A/m, D (D=εH) is the electric flux density in Coulomb/m^2, B (B=μH) is the magnetic flux density in Wb/m^2, M (M=$\sigma^* H$)is the (fictitious) equivalent magnetic current density in V/m^2 and the J (M=σE) is the current density in A/m^2. By handling equations (1a-1d), it is possible to obtain two differential equations in partial derivatives, one for the electric field another for magnetic field [4]:

$$\nabla^2 \mathrm{E} = \mu\sigma \frac{\partial \mathrm{E}}{\partial \mathrm{t}} + \mu\varepsilon \frac{\partial^2 \mathrm{E}}{\partial \mathrm{t}^2} \tag{2a}$$

$$\nabla^2 \mathrm{H} = \mu\sigma \frac{\partial \mathrm{H}}{\partial \mathrm{t}} + \mu\varepsilon \frac{\partial^2 \mathrm{H}}{\partial \mathrm{t}^2} \tag{2b}$$

These two differential equations are known as wave equations for a material medium. It is worth noting that, although we have obtained a wave equation for the electric field Eand another for the magnetic field H, the solution of both equations are not independent, because the electric and magnetic fields are related through Maxwell's equations (2a) and (2b) respectively [4]. A perfect dielectric medium is defined as a material in which the conductivity is $\sigma = 0$. in this category fall most of the substrate materials used for integrated optical devices, such as glasses, faro- electric crystals or polymers, while metals do not belong to this category because of their high conductivity. Then, for dielectric media ($\sigma = 0$) the wave equations simplify on the forms:

$$\nabla^2 \mathrm{E} = \mu\varepsilon \frac{\partial^2 \mathrm{E}}{\partial \mathrm{t}^2} \tag{3a}$$

$$\nabla^2 \mathrm{H} = \mu\varepsilon \frac{\partial^2 \mathrm{H}}{\partial \mathrm{t}^2} \tag{3b}$$

The solution of these equations represent wave that propagate with speed, v, given by $v = \frac{1}{\sqrt{\mu\varepsilon}}$[4].

2.2 Monochromatic plane wave

Since different frequencies in the visible range correspond to different colors, such wave are called monochromatic. Suppose that the waves are travelling in x direction and have no y or z dependence; these called plane waves, because the fields are uniform over every plane perpendicular to the direction of propagation. We are interested then, in the field of the form [4]:

$$E(x,t) = E_0 e^{j(kx - \omega t)} \tag{4a}$$

$$B(x,t) = B_0 e^{j(kx - \omega t)} \tag{4b}$$

Where E_0 , B_0 are the complex amplitude, k is the wave number and ω is the angular frequency [4].

III. Boundary Conditions and Fresnel's Coefficients

At all points on the boundary, normal components of D and B, and tangential component of E and H are continuous. The boundary condition at z=0 are [2]:

$$[(\varepsilon E_0 + E_0'') - \varepsilon' E_0'].n = 0 \quad (5a)$$

$$[K \times E_0 + K'' \times E_0'' - K' \times E_0'].n = 0 \quad (5b)$$

$$[E_0 + E_0'' - E_0'] \times n = 0 \quad (5c)$$

$$\left[\frac{1}{\mu}(K \times E_0 + K'' \times E_0'') - \frac{1}{\mu'} K' \times E_0'\right] \times n = 0 \quad (5d)$$

By applying the boundary conditions, it is convenient to consider two separate situations, the incident plane wave is linearly polarized with its polarization vector (a) perpendicular (TE - mode) and (b) parallel (TM - mode) to the plane of incident [2].

IV. Perpendicular Polarization (TE – mode)

In the perpendicular polarization (TE-mode) case the incident electric field , $\overrightarrow{E_i}$ is directed out of the page in a_y direction [5].

With respect to the considered geometry shown in Fig.1, the incident electric field, as a complex phasor notation, is expressed as [5]

$$\overrightarrow{E_i} = a_y E_{0i} e^{-jk_i.\vec{r}} \quad (6)$$

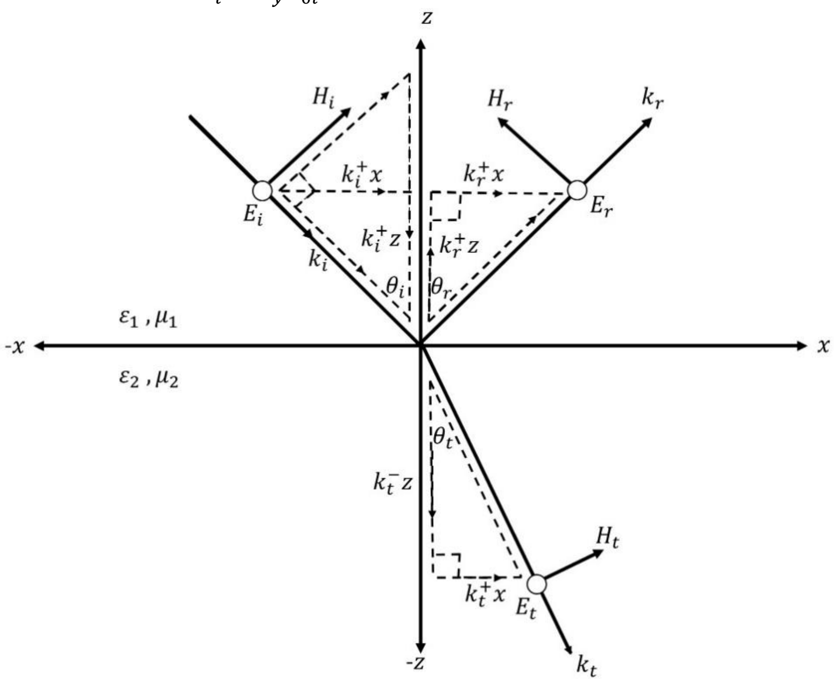


Fig.2: Considered geometry plane wave in the case of TE – mode with incident angle θ_i

The incident wavenumber in the upper half-space $\overrightarrow{k_i}$ is given by [5]

$$\overrightarrow{k_i} = \begin{bmatrix} a_x & a_y & a_z \\ k_1 \sin\theta_i & 0 & k_1 \cos\theta_i \end{bmatrix} \quad (7)$$

where $k_1 = \omega\sqrt{\varepsilon_1 \mu_1}$ is the magnitude of the wavenumber for the upper half-space. The corresponding magnetic field may be written as [5]

$$\overrightarrow{H_i} = \frac{\overrightarrow{k_i} \times \overrightarrow{E_i}}{\omega\mu_1} = \frac{E_{0i}}{\eta_1}(-a_x cos\theta_i + a_z sin\theta_i) e^{-jk_i.\vec{r}} \quad (8)$$

where $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$ is the intrinsic wave impedance for the first medium. Upon reaching the boundary at the z=0 plane. Part of the wave will be reflected and part will be transmitted or reflected. The corresponding expression for these field quantities are given by [5]

$$\overrightarrow{E_r} = a_y E_{0r} e^{-jk_i.\vec{r}} \quad (9)$$

and

$$\overrightarrow{H_r} = \frac{\overrightarrow{k_r} \times \overrightarrow{E_r}}{\omega\mu_1} = \frac{E_{0r}}{\eta_1}(a_x cos\theta_r + a_z sin\theta_r) e^{-jk_i.\vec{r}} \quad (10)$$

where

$$\overrightarrow{k_r} = \begin{bmatrix} a_x & a_y & a_z \\ k_1 \sin\theta_r & 0 & -k_1 \cos\theta_r \end{bmatrix} \tag{11}$$

At this point, the total field quantities in the upper half-space can be written as [5]

$$\overrightarrow{E_1} = \overrightarrow{E_\iota} + \overrightarrow{E_r} \tag{12}$$

and

$$\overrightarrow{H_1} = \overrightarrow{H_\iota} + \overrightarrow{H_r} \tag{13}$$

The transmitted field quantities can be written as [5]

$$\overrightarrow{E_t} = a_y E_{0t} e^{-jk_t.\vec{r}} \tag{14}$$

where

$$\overrightarrow{k_t} = \begin{bmatrix} a_x & a_y & a_z \\ k_2 \sin\theta_t & 0 & k_2 \cos\theta_t \end{bmatrix} \tag{15}$$

With $k_2 = \omega\sqrt{\varepsilon_2\mu_2}$, the magnetic field intensity is given by

$$\overrightarrow{H_t} = \frac{\overrightarrow{k_t} \times \overrightarrow{E_t}}{\omega\mu_2} = \frac{E_{0t}}{\eta_2}(-a_x cos\theta_t + a_z sin\theta_t)e^{-jk_r.\vec{r}} \tag{16}$$

Where $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$ is the intrinsic wave impedance for the second medium. imposing the boundary conditions at the interface z=0 requires the continuity of the tangential components of the electric and magnetic field intensities. The first of these conditions leads to [5]

$$\left.\overrightarrow{E_\iota}\right|_{z=0}.a_y = \left.\overrightarrow{E_t}\right|_{z=0}.a_y \tag{17}$$

or

$$E_{0i}e^{-jk_1 sin\theta_i x} + E_{0r}e^{-jk_1 sin\theta_r x} = E_{0t}e^{-jk_2 sin\theta_t x} \tag{18}$$

And this should be valid irrespective of the value of x, i.e... the following should be satisfied [5]

$$k_1 sin\theta_i = k_1 sin\theta_r = k_2 sin\theta_t \tag{20}$$

This condition is commonly referred to as phase matching and leads to the following Snell's low of reflection and refraction [5]

$$\theta_i = \theta_r \tag{21}$$

$$k_1 sin\theta_i = k_2 sin\theta_t \tag{22}$$

Upon using equation (18), equation (20) can be rewritten as [5]

$$1 + \frac{E_{0r}}{E_{0i}} = \frac{E_{0t}}{E_{0i}} \tag{23}$$

The appropriate ratio of the electric field dente the Fresnel reflection and transmission coefficients as defined respectively as [5]

$$\mathrm{R}_{TE} = \frac{E_{0r}}{E_{0i}} \tag{24}$$

$$\mathrm{T}_{TE} = \frac{E_{0t}}{E_{0i}} \tag{25}$$

In order to write an explicit expression for these two coefficients, the other boundary condition for the magnetic field intensity is applied yielding to [5]

$$\left.\overrightarrow{H_\iota}\right|_{z=0}.a_x = \left.\overrightarrow{H_t}\right|_{z=0}.a_x \tag{26}$$

or

$$\frac{cos\theta_i}{\eta_1}(1 - \mathrm{R}_{TE}) = \frac{cos\theta_t}{\eta_2}(\mathrm{T}_{TE}) \tag{27}$$

Where substitution from equation (20) and equation (21) have been made to write equation (27). Solving equation (23) with (27) to obtain [5]

$$\mathrm{R}_{TE} = \frac{\eta_2 cos\theta_i - \eta_1 cos\theta_t}{\eta_2 cos\theta_i + \eta_1 cos\theta_t} \tag{28}$$

$$\mathrm{T}_{TE} = \frac{2\eta_2 cos\theta_i}{\eta_2 cos\theta_i + \eta_1 cos\theta_t} \tag{29}$$

From these two equations, remarkable expression should be stated. The first is concerned with the exponential factor $\overrightarrow{H_t}$ witch may be written explicitly as [5]

$$e^{jk_2 \sin\theta_t x} e^{jk_2 cos\theta_t z} \tag{30}$$

The first factor represents plane wave propagating in the x- direction, which is form equation (20) and equals to [5]

$$e^{jk_2 \sin\theta_t x} \tag{31}$$

The second factor, however, needs some caution since

$$k_2 cos\theta_i = k_2\sqrt{1 - sin^2{}_{\theta_t}} = k_2\sqrt{1 - (\frac{k_1}{k_2} sin_{\theta_i})^2} \tag{32}$$

Which for nonmagnetic media can be written as

$$k_2 cos\theta_i = k_2 \sqrt{1 - (\frac{n_1}{n_2} sin_{\theta_i})^2} \quad (33)$$

where $n_j = \sqrt{\varepsilon_j}$, the refractive index of the j[th] medium. This factor becomes purely imaginary whenever $\frac{n_1}{n_2} sin_{\theta_i} > 1$ and leads to an evanescent wave. This is commonly referred to as total internal reflection, and has a wide application in optical fiber domain [6]. Another point of particular interest is to obtain a condition that enable the wave to proceed without any reflection (totally transmitted). This may be found by equating to numerator in equation (28) to zero which leads, for nonmagnetic materials, to [5]

$$\frac{cos\theta_i}{cos\theta_t} = \frac{n_2}{n_1} = \frac{sin\,\theta_i}{sin\,\theta_t} \quad (34)$$

This is an impossible condition unless $n_1 = n_2$, where the wave is propagating in the same medium [5]

V. Parallel Polarization (TM – mode)

In the parallel polarization case, the duality between the field quantities is treated and we prefer to sketch the development in order to raise some hidden points and to show that the same general procedure may be carried out to derive the required coefficients. In this case, the incident magnetic field , $\overrightarrow{H_i}$ is directed out of the page in a_y direction [5].With respect to the considered geometry shown in Fig.2 and by using a similar notation, as [5].

$$\overrightarrow{H_i} = a_y H_{0i} e^{-jk_i.\vec{r}} \quad (35)$$

$$\overrightarrow{E_i} = \frac{\overrightarrow{H_i} \times \overrightarrow{k_i}}{\omega\varepsilon_1} = \eta_1 H_{0i}(a_x cos\theta_i - a_z sin\theta_i)e^{-jk_i.\vec{r}} \quad (36)$$

$$\overrightarrow{H_r} = -a_y E_{0r} e^{-jk_i.\vec{r}} \quad (37)$$

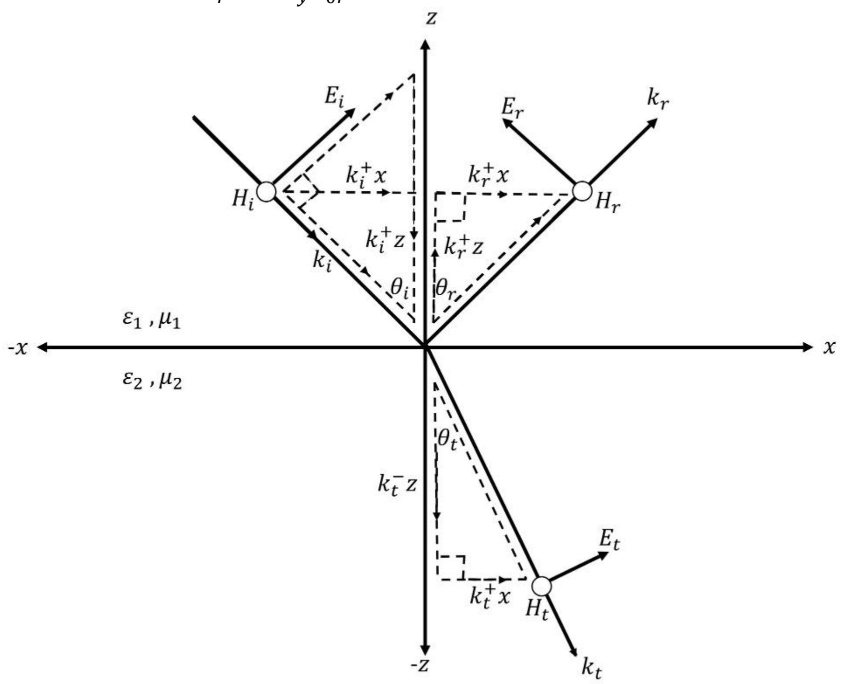


Fig.3: Considered geometry plane wave in the case of TM – mode with incident angle θ_i

$\overrightarrow{E_r} = \eta_1 H_{0r}(a_x cos\theta_r + a_z sin\theta_r)e^{-jk_r.\vec{r}}$	(38)

$$\overrightarrow{H_t} = a_y E_{0t} e^{-jk_t.\vec{r}} \quad (39)$$

$$\overrightarrow{E_t} = \eta_2 H_{0t}(a_x cos\theta_t - a_z sin\theta_t)e^{-jk_t.\vec{r}} \quad (40)$$

By means of Imposing the boundary condition through equations (20) and (21) respectively, the following equations governing the field's ratio can be derived as following [5]

$$1 - \frac{H_{0r}}{H_{0i}} = \frac{H_{0t}}{H_{0i}} \quad (41)$$

$$Z_1 \cos\theta_i \left(1 + \frac{H_{0r}}{H_{0i}}\right) = Z_2 \cos\theta_t \frac{H_{0t}}{H_{0i}} \quad (42)$$

Solving these two equations yield to

$$\frac{H_{0r}}{H_{0i}} = \frac{Z_2 \cos\theta_t - Z_1 \cos\theta_i}{Z_2 \cos\theta_t + Z_1 \cos\theta_i} \quad (43)$$

$$\frac{H_{0t}}{H_{0i}} = \frac{2Z_1 \cos\theta_i}{Z_2 \cos\theta_t + Z_1 \cos\theta_i} \quad (44)$$

These ratios may be defined as was done in equations (24) and (25) but with respect to magnetic field intensities. This is what usually done in transmission line theory to define a voltage and/or current reflection and transmission coefficients [5]. The preferred choice here is to stay with the previously defined ratios as Fresnel coefficients. The field's amplitudes are related via the intrinsic wave impedance as [5]

$$H_{0i} = \frac{E_{0i}}{Z_1} \quad (45)$$

$$H_{0r} = \frac{E_{0r}}{Z_1} \quad (46)$$

$$H_{0t} = \frac{E_{0t}}{Z_2} \quad (47)$$

Substituting in equations (43) and (44) and using the definition in equations (24) and (25), but for parallel polarization we obtain

$$\mathrm{R}_{TM} = \frac{Z_2 \cos\theta_t - Z_1 \cos\theta_i}{Z_2 \cos\theta_t + Z_1 \cos\theta_i} \quad (48)$$

$$\mathrm{T}_{TM} = \frac{2Z_1 \cos\theta_i}{Z_2 \cos\theta_t + Z_1 \cos\theta_i} \quad (49)$$

It is of interest, however, to note that at normal incidence ($\theta_i = 0$)the reflection and the transmission coefficients are related as may be deduced from equation (42)

$$1 + \mathrm{R}_j = \mathrm{T}_j \quad (50)$$

Where j= TE, TM mode. As suggested by equation (23) and this is true for all incident angle. The critical angle at which the wave may be proceed without reflection, for this type of polarization, is designated by so called Brewster's angle. Setting the numerator of equation (48) to zero will lead, upon using equation (22) for nonmagnetic media to

$$\theta_B = tan^{-1}(^{n_1}/_{n_2} \quad (51)$$

VI. Results

This section presents the work accomplished in studying the behavior of an arbitrarily polarized plane wave on a dielectric-dielectric interface. All simulations have been carried out using self-built MATLAB code which is used to construct the structure and post-process the obtained data.

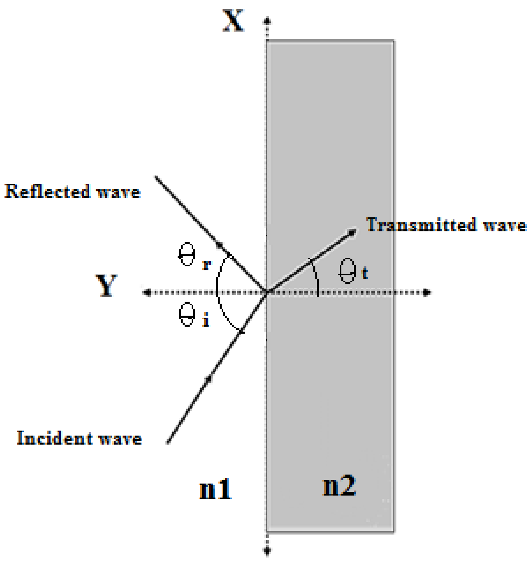


Fig.4: planar interface between two media

The first model analyzed is two-dimension planar interface between two media depicted in figure 4, with medium 1 and medium 2 refractive indices of n1 and n2 respectively.
The variation of reflection and transmission coefficients with incident angle for TM case is illustrated in figures 5

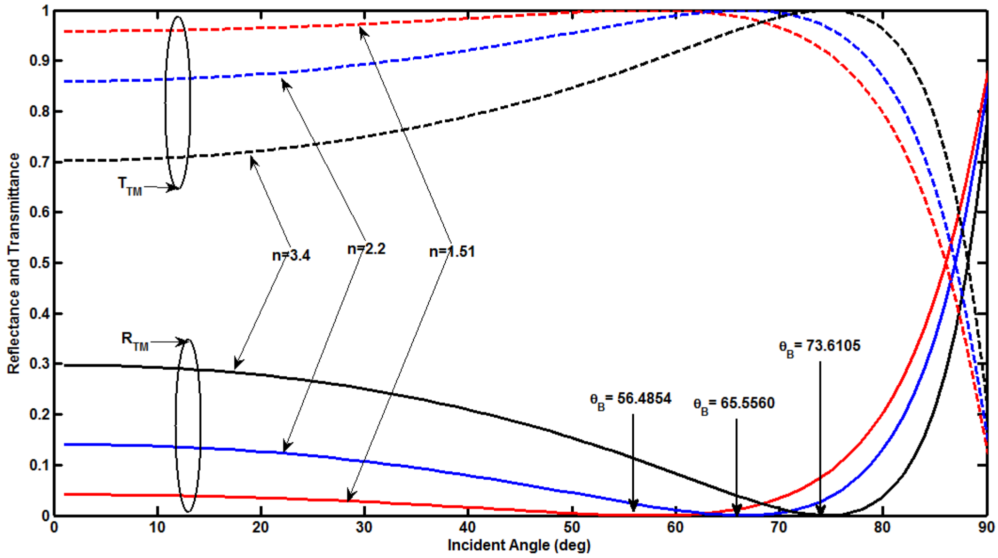


Fig.5:Reflectance and transmittance for TM incidence corresponding to n1=1 and n2= 1.51, 2.2, and 3.4 respectively.

Figure 5 shows the reflectance and transmittance coefficients as a function of the incident angle in case of air- glass (red), air-Gallium arsenide (blue), and air-Lithium niobate (black) where the refractive indices are 1.51, 2.2, and 3.4 respectively. As may be observed from figure 3.2, the Brewster's angles are about 56.48, 65.55, and 73.61 respectively. It is clear that the increase of refractive index will lead in turn to increase in the Brewster's angle. Figure 5 shows that the reflectance around the Brewster's angle is close to zero which means that the transmitted wave suffers from bending or refraction at the interference. In addition to that, for the case of air-glass the reflectance at normal incidence is ≈ 4%, which is relatively low value. Nevertheless, for materials with higher refractive indices ($LiNbO_3$, n=2.2; GaAs, n=3.4) the reflectance at normal incidence is higher enough ($LiNbO_3$=14%, GaAs= 30%) to be used as partially reflecting mirrors in some integrated photonic components.

The reflectance and transmittance of TE-polarized wave incident from air to glass (red), air to Gallium arsenide (blue), and air to Lithium niobate (black) are plotted in figure 6.

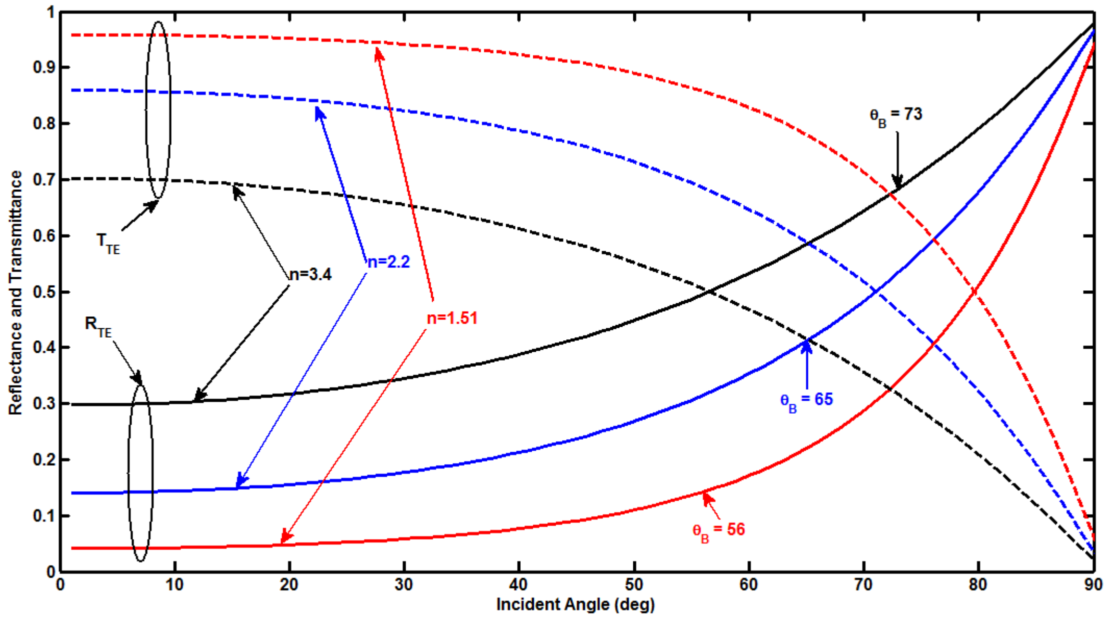


Fig.6:Reflectance and transmittance for TE incidence corresponding to n1=1 and n2= 1.51, 2.2, and 3.4

respectively.

At variance to that found in TM incidence, in TE incidence the reflectance is a monotonous increasing function of the incident angle. Therefore, if a beam of non-polarized light is incident at an angle of θ_B , the interface only will reflect the TE component of such radiation, and thus the reflected wave will be linearly polarized with the electric field vector perpendicular to the incident plane. This is the reason why Brewster's angle is also called the polarizing angle, and this phenomenon can be used to design polarization devices. A graph of the reflectivity for the air-glass interface as a function of the angle of incidence is shown in figure 7.

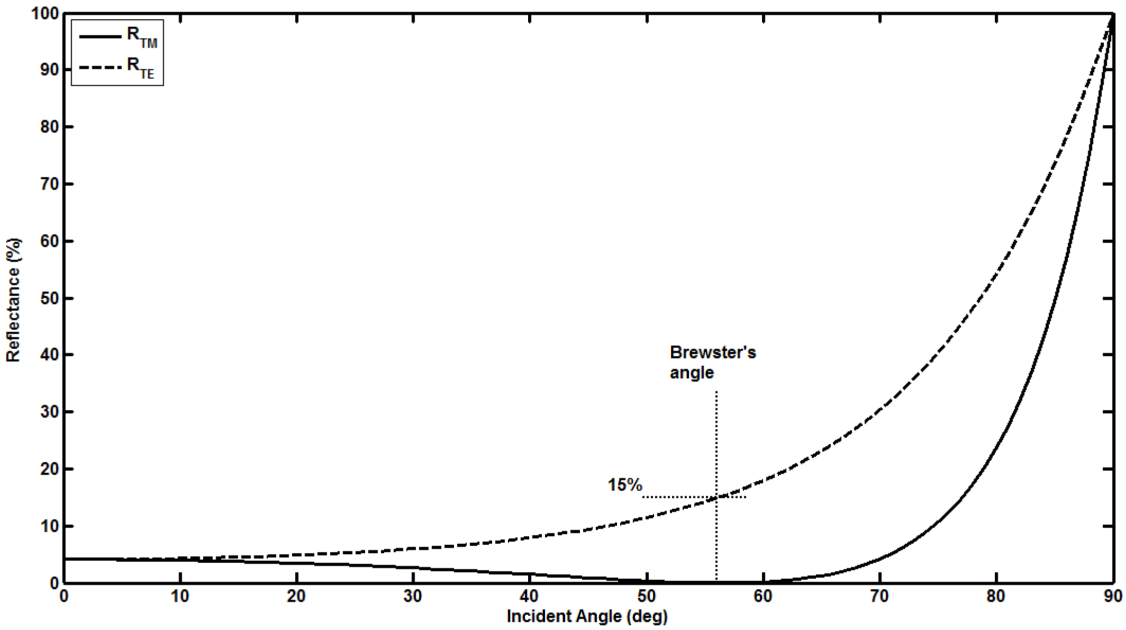


Fig.6: the air-glass reflectance as a function of the angle of incidence

At normal incidence, the parallel (TM) and perpendicular (TE) polarization waves are physically identical and have the same reflectivity. Notice that even though the glass is transparent, 4% of the light is still reflected. However, as angle of incidence increases, the parallel component drops and the perpendicular component rises, until at the Brewster's angle their values are approximately 0% and 15 % of the initial intensity of the light respectively. At an incident angle of $90^o$the entire incident light is reflected, so the substance acts as a mirror.

The magnitudes of the phase shifts play an essential role in establishing the condition of wave propagation in planar optical waveguides, which gives rise to the calculation of the allowed propagation modes, as we will see in the next section. These phase shifts are represented in figure 7, for the case of total internal reflection in the boundary glass-air (n_1=1.51 and n_2=1).

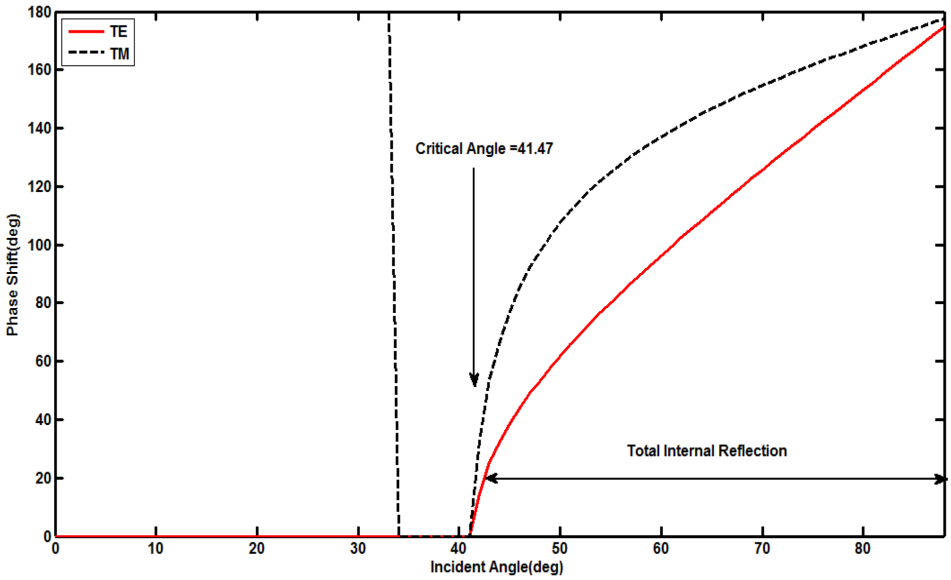


Fig.7:phase shift experienced by the reflected waves for TM and TE incidence produced

VII. Conclusion

This paper addresses the reflection and transmission of plane wave obliquely incident on two dielectric media. The primary objectives of this paper were to develop and refine MATLAB algorithms for modelling an electromagnetic monochromatic plane wave travelling through a homogeneous medium, incident on a second homogenous medium, separated from the former by a planar interface. During the course of this research, Fresnel transmission and reflection coefficients in conjunction with Snell's low were developed for studying the behavior of arbitrary polarized plane wave on a dielectric-dielectric interface. This paper presented detailed results and key design parameters and comparisons between different dielectric materials.

References

[1] G. Lifante, *Integrated Photonics Fundamentals*, Wiley, 2003.
[2] M. Koshiba, *Optical Waveguide Analysis*, McGraw Hill, 1990
[3] K. Okamoto, *Fundamental of Optical Waveguides*, Elsevier, 2006.
[4] Carl T. A. johnk, Engineering electromagnetic fields and waves second.
[5] Pozar D, Microwave Engineering, John Wily and Aons,1975
[6] Suematsu Y and Iga K, Introduction to optical fiber communications, John Wily and Aons,1982

Nabeil A. Abujnah, et. al. "Analytical modelling of Plane Wave Obliquely Incident on Two Dielectric media." *IOSR Journal of Electronics and Communication Engineering (IOSR-JECE)* 16(1), (2021): 22-30.